CHAPTER 4 - CIRCUIT THEOREMS

List of topics for this chapter :

- Linearity Property
- Principle of Superposition
- Source Transformations
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer
- Verifying Circuit Theorems with PSpice

LINEARITY PROPERTY

Linearity is the condition in which the change in value of one quantity is directly proportional to that of another quantity. A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Problem 4.1 If all the independent sources are multiplied by a value, K, and all the currents and voltages of the circuit increase by the same value, then a circuit is linear. Show that the circuit in Figure 4.1 is linear by solving for I_x and V_x.

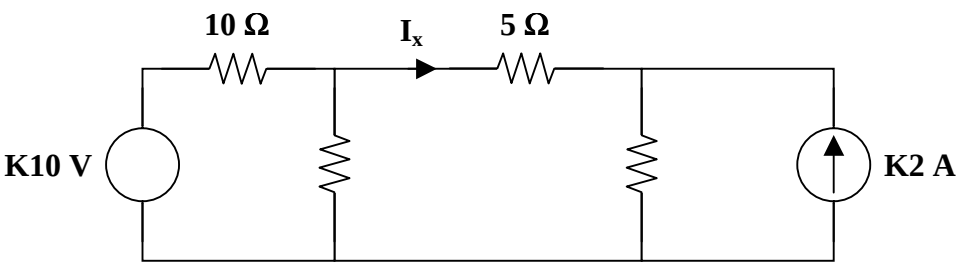
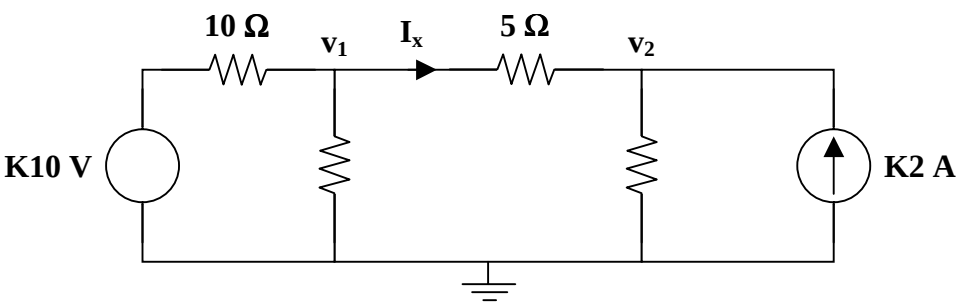


Figure 4.1



Use nodal analysis to find v_1 and v_2.

At node 1 :

$$\frac{v_1 - K10}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

Simplifying,

$$v_1 - K10 + v_1 + (2)(v_1 - v_2) = 0$$

$$4v_1 - 2v_2 = K10$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = K2$$

$$v_2 - v_1 + v_2 = K10$$

$$-v_1 + 2v_2 = K10$$

The system of simultaneous equations is

$$\begin{bmatrix} 4 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} K10 \\ K10 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{8-2} \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} K10 \\ K10 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} K20 + K20 \\ K10 + K40 \end{bmatrix} = \begin{bmatrix} (40/6)K \\ (50/6)K \end{bmatrix} = \begin{bmatrix} (20/3)K \\ (25/3)K \end{bmatrix}$$

Clearly, $V_x = v_1 - v_2 = (20/3)K - (25/3)K = (-5/3)K$

and $I_x = V_x/5 = [(-5/3)K](1/5) = (-1/3)K$

The voltage source and current source are multiples of K. The voltage V_x and the current I_x are multiples of K. Therefore, **the circuit is linear**.

Problem 4.2 *[4.3]*

(a) In the circuit in Figure 4.1, calculate v_o and i_o when $v_s = 1\text{ V}$.

(b) Find v_o and i_o when $v_s = 10\text{ V}$.

(c) What are v_o and i_o when each of the 1-Ω resistors is replaced by a 10-Ω resistor and $v_s = 10\text{ V}$?

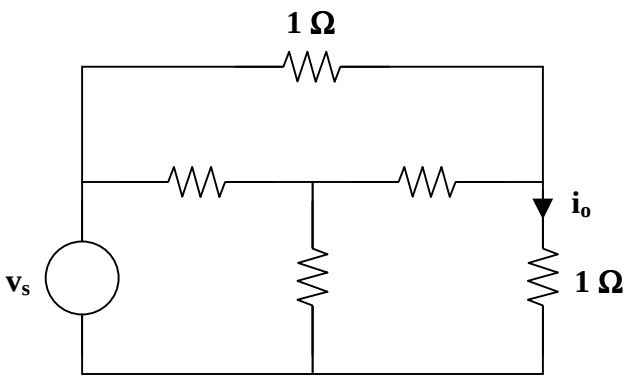
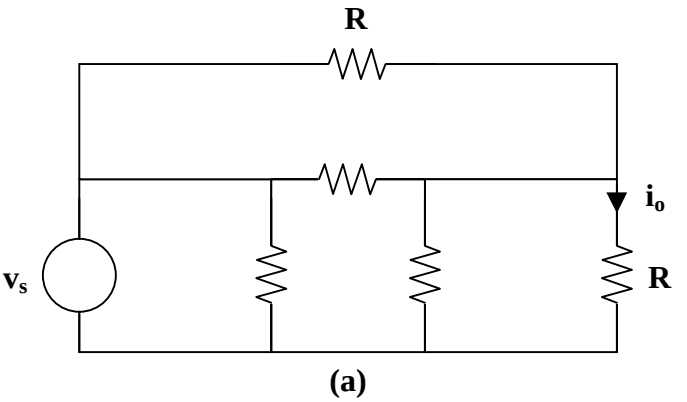
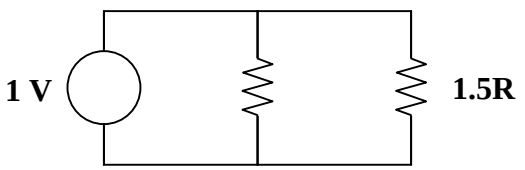


Figure 4.1

First, transform the Y (or T) subcircuit to its Δ (or Π) equivalent.



(a)



(b)

$$R \parallel 3R = \frac{3R^2}{4R} = \frac{3}{4}R \qquad \frac{3}{4}R + \frac{3}{4}R = \frac{3}{2}R$$

$v_o = v_s/2$, independent of R

and

$i_o = v_o/R$

(a) When $v_s = 1$ V and R = 1 Ω,

$v_o =$ **0.5 V** $\qquad i_o =$ **0.5 A**

(b) When $v_s = 10$ V and R = 1 Ω,

$v_o =$ **5 V** $\qquad i_o =$ **5 A**

(c) When $v_s = 10$ V and $R = 10\,\Omega$,

$v_o =$ **5 V** $\qquad i_o =$ **0.5 A**

PRINCIPLE OF SUPERPOSITION

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Problem 4.3 Solve for I_x in Figure 4.1 using superposition.

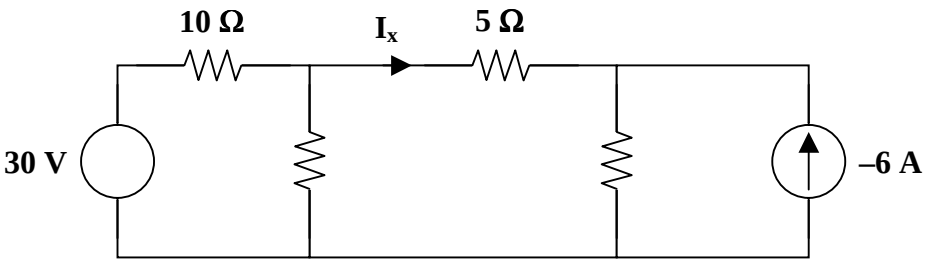


Figure 4.1

- **Carefully DEFINE the problem.**
  Each component is labeled completely. The problem is clear.

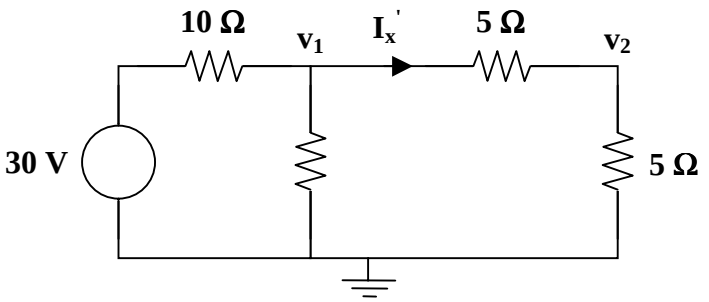
- **PRESENT everything you know about the problem.**
  Using the principle of superposition, we will need to find the desired current when the current source is turned off, or set equal to zero. This implies the replacement of the current source with an open circuit. Let this value be equal to I_x'. We will also need to find the desired current when the voltage source is turned off. This implies the replacement of the voltage source with a short circuit. Let this value be equal to I_x''. Then, the desired current is the sum of these two currents, i.e.,

$$I_x = I_x' + I_x''$$

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  This circuit, having two independent sources, can be analyzed using nodal or mesh analysis, as seen in Chapter 3. The problem statement requires the use of superposition. The principle of superposition can be very useful, especially when there are several independent sources. More work may be required to find the desired voltage or current, but the analysis is performed using simpler circuits.

➢ **ATTEMPT a problem solution.**

Setting the current source to zero, the circuit becomes



Use nodal analysis to find v_1 and v_2.

At node 1 :

$$\frac{v_1 - 30}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = 0$$

Simplifying,

$$v_1 - 30 + v_1 + (2)(v_1 - v_2) = 0$$

$$4v_1 - 2v_2 = 30$$

$$2v_1 - v_2 = 15$$

$$v_2 - v_1 + v_2 = 0$$

$$-v_1 + 2v_2 = 0$$

The system of simultaneous equations is

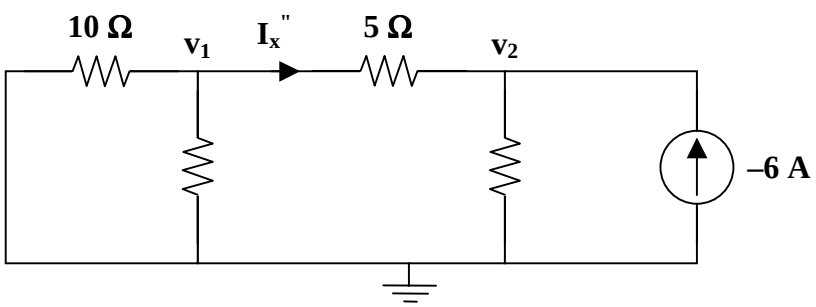
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{4-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 15 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 30 \\ 15 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Clearly, $V_x' = v_1 - v_2 = 10 - 5 = 5$ volts.

and $I_x' = \frac{V_x'}{5} = \frac{5}{5} = 1$ amp.

Setting the voltage source to zero, the circuit becomes



Use nodal analysis to find v_1 and v_2.

At node 1 :

$$\frac{v_1 - 0}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = -6$$

Simplifying,

$$v_1 + v_1 + (2)(v_1 - v_2) = 0 \qquad v_2 - v_1 + v_2 = -30$$
$$4v_1 - 2v_2 = 0 \qquad -v_1 + 2v_2 = -30$$
$$2v_1 - v_2 = 0$$

The system of simultaneous equations is

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{4-1} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ -30 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -30 \\ -60 \end{bmatrix} = \begin{bmatrix} -10 \\ -20 \end{bmatrix}$$

Clearly, $V_x'' = v_1 - v_2 = -10 - (-20) = 10$ volts.

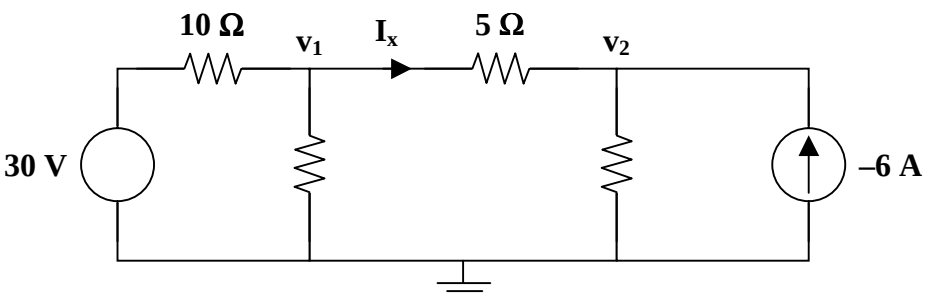
and $I_x'' = \frac{V_x''}{5} = \frac{10}{5} = 2$ amps.

The sum of the currents, I_x' and I_x'', due to the two independent sources acting alone is the current, I_x, of the circuit due to both sources.

$$I_x = I_x' + I_x''$$
$$I_x = 1 + 2 = 3 \text{ amps.}$$

➢ **EVALUATE the solution and check for accuracy.**

Find I_x using both sources and nodal analysis. The circuit is as follows.



At node 1 :

$$\frac{v_1 - 30}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = -6$$

Simplifying,

$$v_1 - 30 + v_1 + (2)(v_1 - v_2) = 0 \qquad v_2 - v_1 + v_2 = -30$$
$$4v_1 - 2v_2 = 30 \qquad -v_1 + 2v_2 = -30$$
$$2v_1 - v_2 = 15$$

The system of simultaneous equations is

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 15 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{4-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 15 \\ -30 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 30-30 \\ 15-60 \end{bmatrix} = \begin{bmatrix} 0 \\ -15 \end{bmatrix}$$

Clearly, $V_x = v_1 - v_2 = 0 - (-15) = 15$ volts.

and $I_x = \dfrac{V_x}{5} = \dfrac{15}{5} = 3$ amps.

This answer is the same as the answer obtained using the principle of superposition. Our check for accuracy was successful.

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.** This problem has been solved satisfactorily.

$$I_x = \underline{\mathbf{3\ A}}$$

Problem 4.4 *[4.11]* Apply the superposition principle to find v_o in the circuit of Figure 4.1.

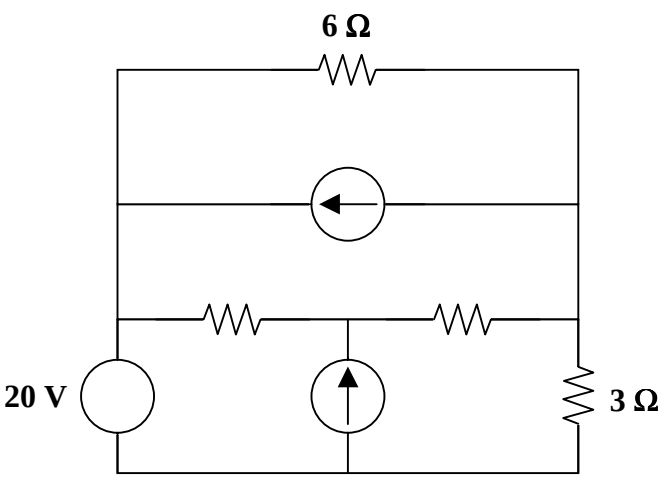
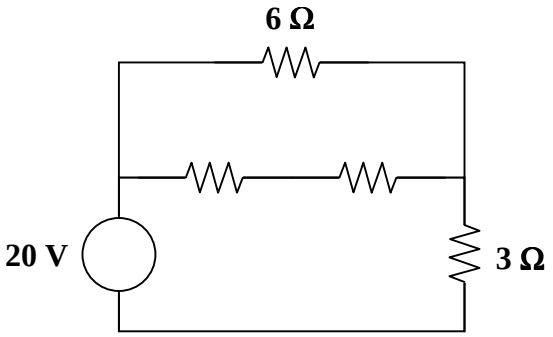


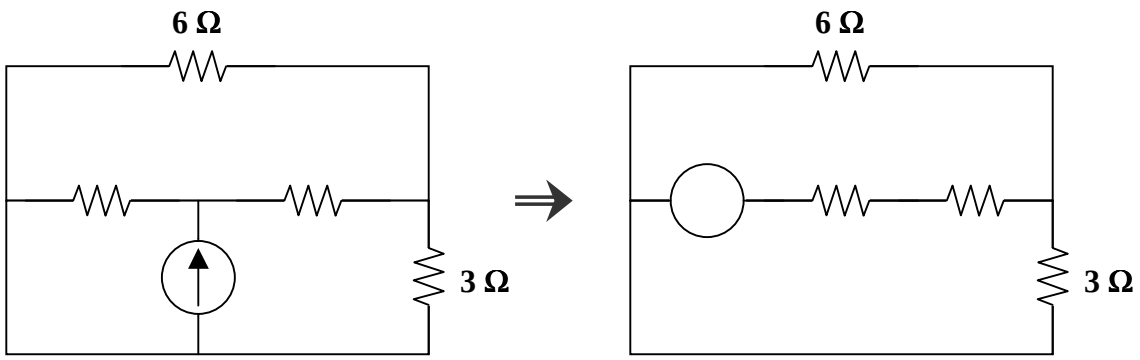
Figure 4.1

Let $v_o = v_{o1} + v_{o2} + v_{o3}$, where v_{o1}, v_{o2}, and v_{o3} are due to the 20-V, 1-A, and 2-A sources, respectively. For v_{o1}, consider the circuit below.



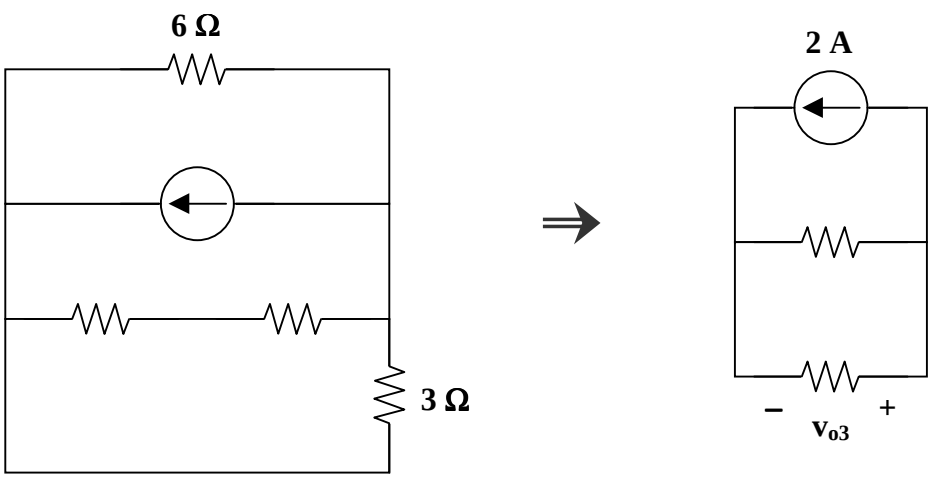
$$6 \,\|\, (4+2) = 3\,\Omega \qquad\qquad v_{o1} = (1/2)(20) = 10 \text{ V}$$

For v_{o2}, consider the circuit below.



$$3 \,\|\, 6 = 2\,\Omega \qquad\qquad v_{o2} = \left(\frac{2}{4+2+2}\right)(4) = 1 \text{ V}$$

For v_{o3}, consider the circuit below.



$$6 \,\|\, (4+2) = 3\,\Omega \qquad\qquad v_{o3} = (-1)(3) = -3$$

Thus,

$$v_o = 10 + 1 - 3 = \underline{\mathbf{8\ V}}$$

Problem 4.5 A linear circuit has the following independent sources: V_1, V_2, and I_S along with a current, I_R, through a resistor. It cannot be assumed that these are the only elements (resistors and dependent sources) in the circuit. Complete the following table.

Circuit number	V_1 (volts)	V_2 (volts)	I_S (amps)	I_R (amps)
Condition #1	10	0	0	2
Condition #2	0	5	0	–1
Condition #3	0	0	3	1
Condition #4	10	5	3	
Condition #5	10	–20	3	
Condition #6				6

This problem is based on linearity and the principle of superposition.

From the Conditions #1, #2, and #3, we know

- $I_R = 2$ amps when the only contributing independent source is $V_1 = 10$ volts.
- $I_R = -1$ amp when the only contributing independent source is $V_2 = 5$ volts.
- $I_R = 1$ amp when the only contributing independent source is $I_S = 3$ amps.

Condition #4 :

$$I_R = \left(\frac{10}{10}\right)(2) + \left(\frac{5}{5}\right)(-1) + \left(\frac{3}{3}\right)(1) = 2 - 1 + 1 = \underline{\mathbf{2\ A}}$$

Condition #5 :

$$I_R = \left(\frac{10}{10}\right)(2) + \left(\frac{-20}{5}\right)(-1) + \left(\frac{3}{3}\right)(1) = 2 + 4 + 1 = \underline{\mathbf{7\ A}}$$

Condition #6 : The following are only three of an infinite set of solutions.

If only one source is not equal to zero

then $V_1 = ?$ $V_2 = \underline{\mathbf{0}}$ $I_S = \underline{\mathbf{0}}$ $I_R = 6$ $\qquad 6 = \left(\frac{V_1}{10}\right)(2)$ $\qquad V_1 = \underline{\mathbf{30}}$

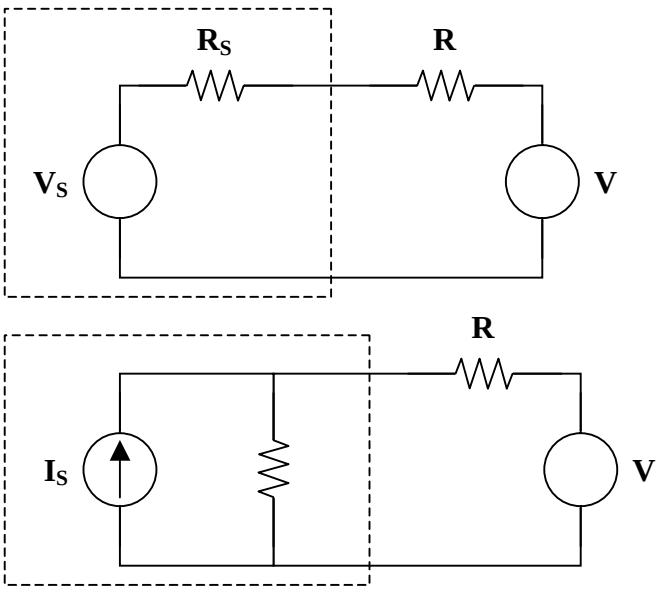
or $V_1 = \underline{\mathbf{0}}$ $V_2 = ?$ $I_S = \underline{\mathbf{0}}$ $I_R = 6$ $\qquad 6 = \left(\frac{V_2}{5}\right)(-1)$ $\qquad V_2 = \underline{\mathbf{-30}}$

or $V_1 = \underline{\mathbf{0}}$ $V_2 = \underline{\mathbf{0}}$ $I_S = ?$ $I_R = 6$ $\qquad 6 = \left(\frac{I_S}{3}\right)(1)$ $\qquad I_S = \underline{\mathbf{18}}$

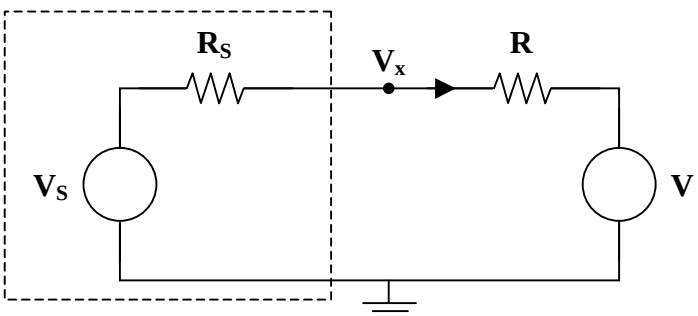
SOURCE TRANSFORMATIONS

A source transformation is the process of replacing a voltage source V_S in series with a resistor R with a current source I_S in parallel with a resistor R or vice versa.

Problem 4.6 Given that the following circuits are linear, prove that the two circuits are equivalent, where $R_S = V_S / I_S$.



Two circuits are said to be equivalent if they have the same voltage-current relationship at their terminals. Begin by finding V_x and I_x in the following circuit.



Clearly,

$$I_x = \frac{V_S - V_x}{R_S}$$

or

$$V_x = V_S - R_S I_x$$

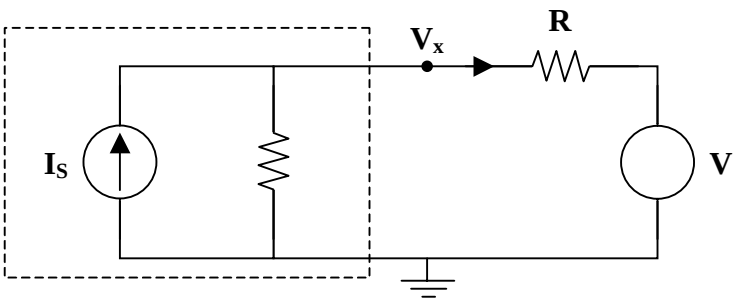
Using KVL,

$$-V_S + (R_S + R)\,I_x + V = 0$$

or

$$I_x = \frac{V_S - V}{R_S + R}$$

Now, find V_x and I_x in this circuit.



Using KCL,

$$I_S = I_x + \frac{V_x}{R_S}$$

or

$$V_x = R_S\,I_S - R_S\,I_x$$

We know that $R_S = \frac{V_S}{I_S}$. So,

$$V_x = V_S - R_S\,I_x.$$

Using mesh analysis,

$$R\,I_x + V + R_S\,(I_x - I_S)$$
$$(R_S + R)\,I_x = R_S\,I_S - V$$
$$I_x = \frac{R_S\,I_S - V}{R_S + R}$$

We know that $R_S = \frac{V_S}{I_S}$. So,

$$I_x = \frac{V_S - V}{R_S + R}.$$

In both circuits, $V_x = V_S - R_S\,I_x$ and $I_x = \frac{V_S - V}{R_S + R}$.

Therefore, **the two circuits are equivalent**.

Problem 4.7 *[4.23]* Given the circuit in Figure 4.1, use source transformation to find i_o.

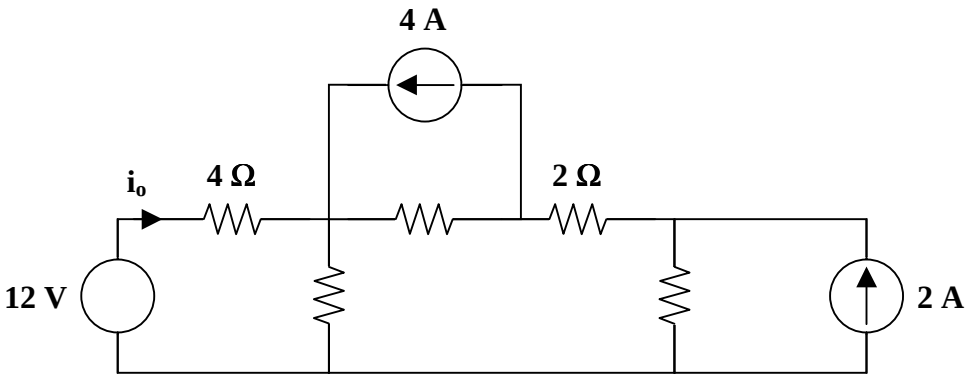
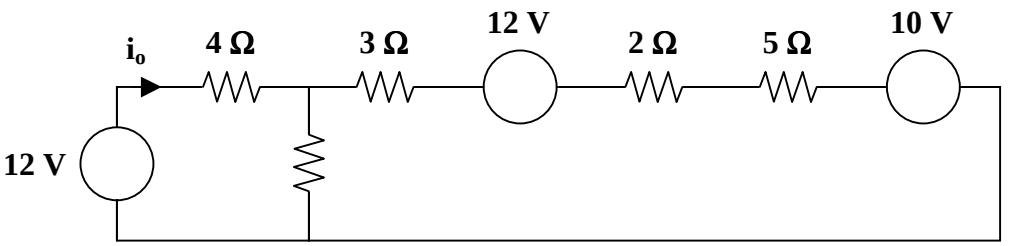
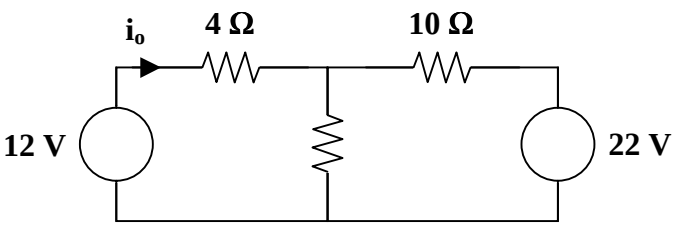


Figure 4.1

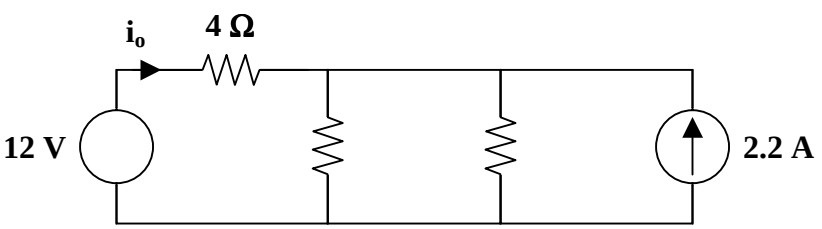
Transforming only the current sources leads to Fig. (a). Continuing with source transformations finally produces the circuit in Fig. (d).



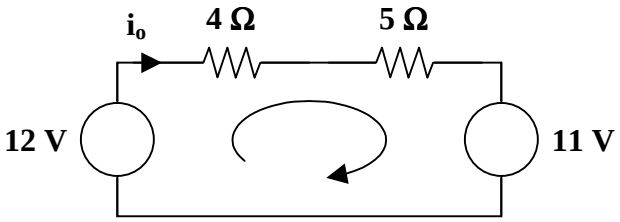
(a)



(b)



(c)



(d)

Applying KVL to the loop in Fig. (d),

$$-12 + 9i_o + 11 = 0 \longrightarrow i_o = 1/9 = \mathbf{111.11\ mA}$$

Problem 4.8 Using source transformations, solve for I_o in Figure 4.1.

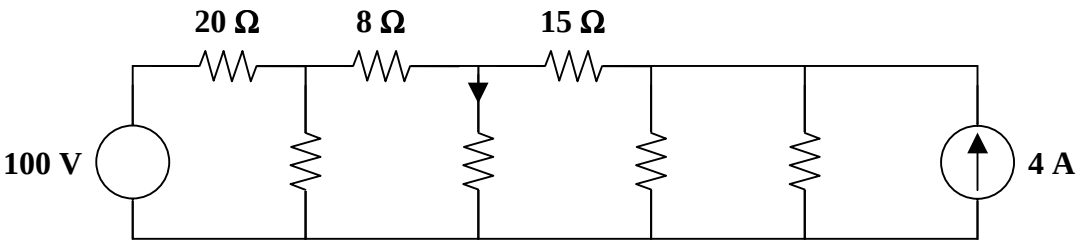
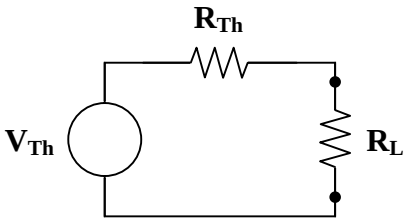


Figure 4.1

$$I_o = \mathbf{2\ A}$$

THEVENIN'S THEOREM

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are zero. An alternative way to find R_{Th} is to find the short-circuit current between the terminals, I_{sc}; then,

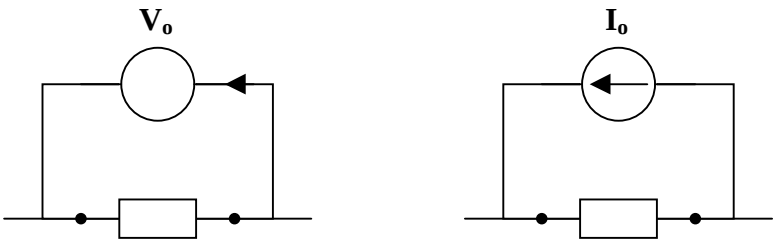


$$V_{Th} = V_{oc}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}}$$

Note that finding the equivalent resistance between the terminals using this approach is a valid method only if there are no dependent sources. Calculating R_{Th} using V_{oc} and I_{sc} is a valid method with independent and dependent sources.

Finally, if $V_{oc} = I_{sc} = 0$, then the circuit must be excited with a voltage source or a current source at the terminals, as shown below, to find R_{Th}.



Find I_o if given a voltage source V_o = 1 V. Find V_o if given a current source I_o = 1 A.
Now, $R_{Th} = 1/I_o$ or $R_{Th} = V_o/1$.

Problem 4.9 Solve for the Thevenin equivalent circuit as seen by the 10-Ω resistor with the current I_o flowing through it in Figure 4.1.

- **Carefully DEFINE the problem.**
  Each component is labeled completely. The problem is clear.

- **PRESENT everything you know about the problem.**
  To find the Thevenin equivalent circuit, we need to find the open-circuit voltage (V_{oc}) across the terminals of the 10-Ω resistor with the current I_o flowing through it. We also need to find the short-circuit current (I_{sc}) through these terminals. Then,

  $$V_{Th} = V_{oc} \qquad \text{and} \qquad R_{Th} = V_{oc} / I_{sc}$$

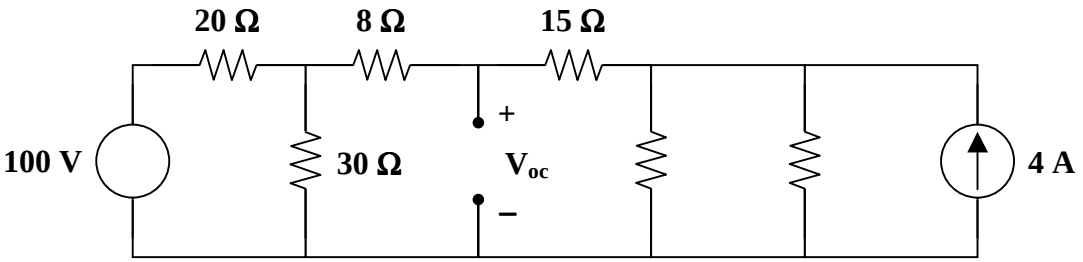
  Because the circuit has no dependent sources, there is an alternative way to find the Thevenin resistance. R_{Th} is the input resistance at the terminals when the independent sources are set equal to zero. This method will not be used in the initial attempt to find a solution. It is better to find the short-circuit current and calculate the Thevenin resistance because it will work for any circuit. However, we can find the input resistance at the terminals to check our initial solution.

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  To find the open-circuit voltage, the obvious techniques are nodal analysis, mesh analysis, or source transformations along with KVL. Because we want to find a voltage, let's eliminate mesh analysis as a promising technique. It is clear that nodal analysis produces a set of three equations and three unknowns. Simplifying the circuit using source transformations yields a single loop from which we can easily find the open-circuit voltage.

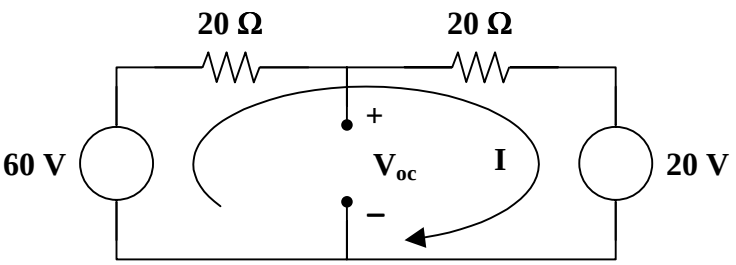
To find the short-circuit current, a similar argument can be made. So, let's eliminate nodal analysis because we are looking for a current rather than a voltage. Mesh analysis gives a set of equations to be solved. Lastly, source transformations allows the circuit to be reduced to a single loop from which we can easily find the short-circuit current.

With the open-circuit voltage and the short-circuit current at the terminals, the Thevenin equivalent circuit is found as stated above.

- **ATTEMPT a problem solution.**
  Begin by finding the open-circuit voltage as seen by the element.



After a couple of source transformations, as seen in Problem 4.8, the circuit becomes



Using KVL, find V_{oc}.

$$-60 + 20I + 20I + 20 = 0$$

$$40I = 40 \longrightarrow I = 1\,\text{amp}$$

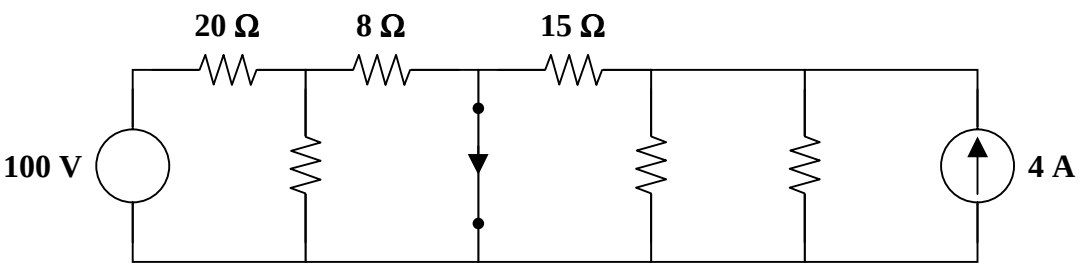
where

$$I = \frac{60 - V_{oc}}{20} = \frac{V_{oc} - 20}{20} = 1$$

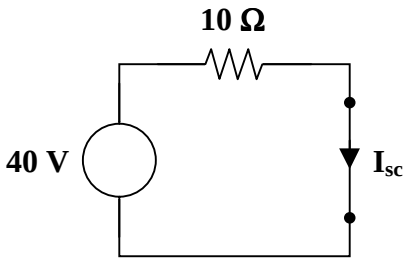
$$60 - V_{oc} = V_{oc} - 20$$

$$V_{oc} = 40\,\text{volts}$$

Now, find the short-circuit current through the element.



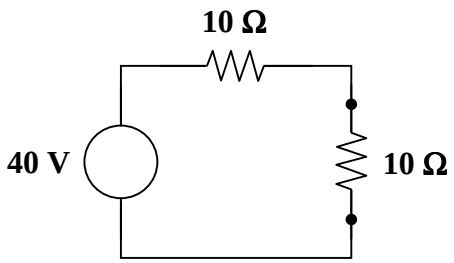
Again using source transformations, the circuit can be reduced as shown below.



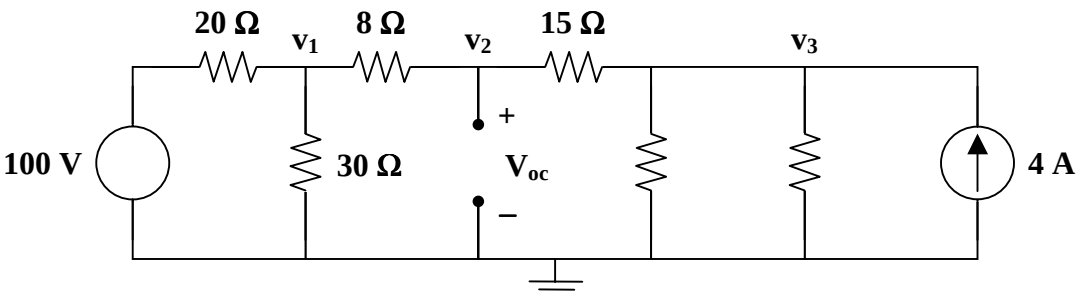
Clearly, $$I_{sc} = \frac{40}{10} = 4 \text{ amps.}$$

Then, $$R_{Th} = \frac{40}{10} = 4 \text{ ohms.}$$

Therefore, the Thevenin equivalent circuit is



➢ **EVALUATE the solution and check for accuracy.**
To check the open-circuit voltage, perform nodal analysis using the circuit below.



For node 1, $$\frac{v_1 - 100}{20} + \frac{v_1}{30} + \frac{v_1 - v_2}{8} = 0$$

For node 2, $$\frac{v_2 - v_1}{8} + \frac{v_2 - v_3}{15} = 0$$

For node 3, $$\frac{v_3 - v_2}{15} + \frac{v_3}{10} + \frac{v_3}{10} - 4 = 0$$

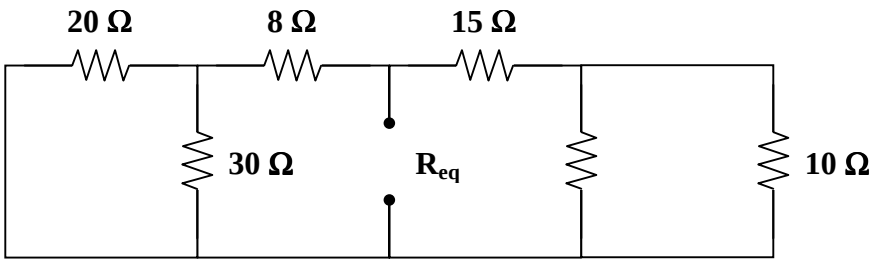
Simplifying these equations and putting them into matrix form results in

$$\begin{bmatrix} 50 & -30 & 0 \\ -15 & 23 & -8 \\ 0 & -2 & 8 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1200 \\ 0 \\ 120 \end{bmatrix}$$

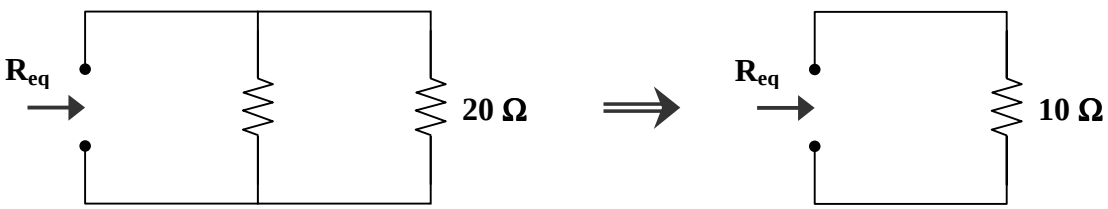
which yields $v_1 = 48$ volts $v_2 = 40$ volts $v_3 = 25$ volts

Hence, $V_{Th} = V_{oc} = v_2 = 40$ volts

Since there are only independent sources, an alternative way to find R_{Th} is to set the sources to zero and calculate the resistance of the modified network. After replacing the voltage source with a short circuit and the current source with an open circuit, the network becomes



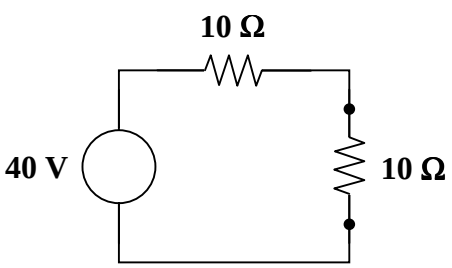
Combine the resistors to the left of the open circuit. (20 Ω || 30 Ω) + 8 Ω = 20 Ω
Combine the resistors to the right of the open circuit. (10 Ω || 10 Ω) + 15 Ω = 20 Ω



An equivalent resistance of 10 Ω matches the value that was calculated using V_{oc} and I_{sc}.

Our check for accuracy was successful.

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
  This problem has been solved satisfactorily. **The Thevenin equivalent circuit is as follows**.



Problem 4.10 Find the Thevenin equivalent as seen by R in Figure 4.1.

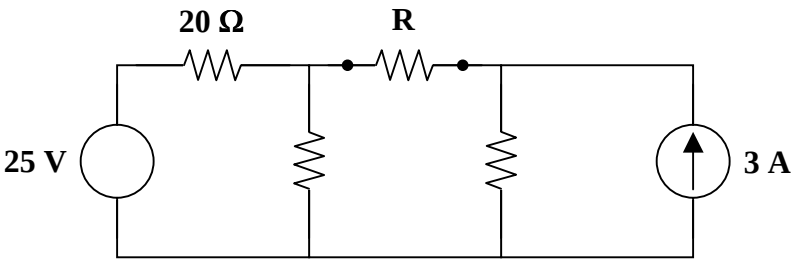
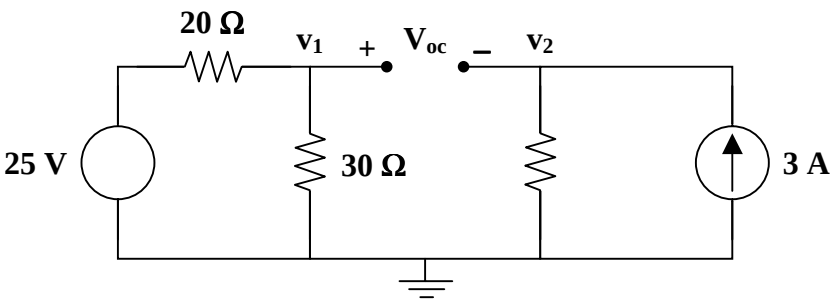


Figure 4.1

First, find the open-circuit voltage, where $V_{Th} = V_{oc}$.



Using nodal analysis,

At node 1 :

$$\frac{v_1 - 25}{20} + \frac{v_1 - 0}{30} = 0$$

$$3v_1 + 2v_1 = 75$$

$$v_1 = 15$$

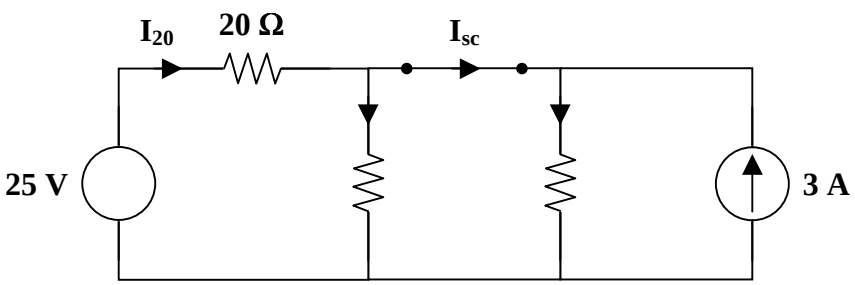
At node 2 :

$$\frac{v_2 - 0}{5} = 3$$

$$v_2 = 15$$

Clearly, $V_{oc} = v_1 - v_2 = 15 - 15 = 0$ volts.

Now, find the short-circuit current.

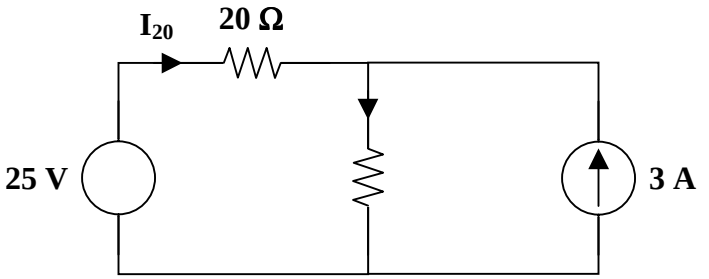


Clearly, $I_{sc} = I_{20} - I_{30}$.

Perform KVL using the left loop.

$$-25 + 20I_{20} + 30I_{30} = 0$$

Begin by finding I_{20} in the following modified circuit.



$$-25 + 20\,I_{20} + \frac{30}{7} I_x = 0 \qquad \text{where } I_x = I_{20} + 3$$

$$140\,I_{20} + (30)(I_{20} + 3) = 175$$

$$170\,I_{20} = 85$$

$$I_{20} = 0.5 \text{ amps}$$

Now,

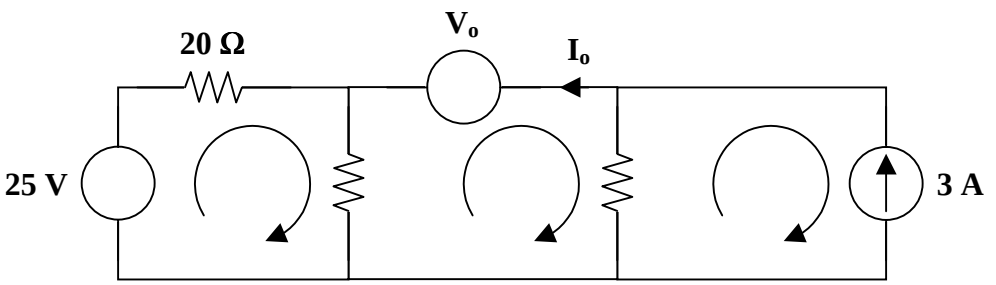
$$-25 + (20)(0.5) + 30\,I_{30} = 0$$

$$I_{30} = 15/30 = 0.5 \text{ amps}$$

Finally,

$$I_{sc} = I_{20} - I_{30} = 0.5 - 0.5 = 0 \text{ amps.}$$

Since 0/0 is undefined, we need to excite the circuit with a 1-V voltage source at the terminals of R in order to find R_{Th}.



Use mesh analysis to find I_o. Then, $R_{Th} = V_o / I_o$.

For loop 1 : $-25 + 20\,i_1 + (30)(i_1 - i_2) = 0$

For loop 2 : $(30)(i_2 - i_1) + V_o + (5)(i_2 - i_3) = 0$ where $V_o = 1$ volt

For loop 3 : $i_3 = -3$ This is the constraint equation.

Simplifying,

$$50\,i_1 - 30\,i_2 = 25$$

$$-30\,i_1 + 35\,i_2 = -16$$

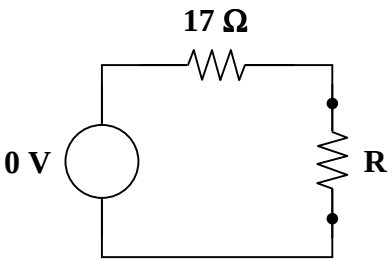
In matrix form,

$$\begin{bmatrix} 50 & -30 \\ -30 & 35 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} 25 \\ -16 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} 0.4647 \\ -0.0588 \end{bmatrix}$$

Now, $I_o = -i_2 = 0.0588$ and $R_{Th} = \dfrac{V_o}{I_o} = \dfrac{1}{0.0588} = 17$ ohms.

Therefore, **the Thevenin equivalent circuit is as follows**.



Problem 4.11 *[4.33]* For the circuit in Figure 4.1, find the Thevenin equivalent between terminals a and b.

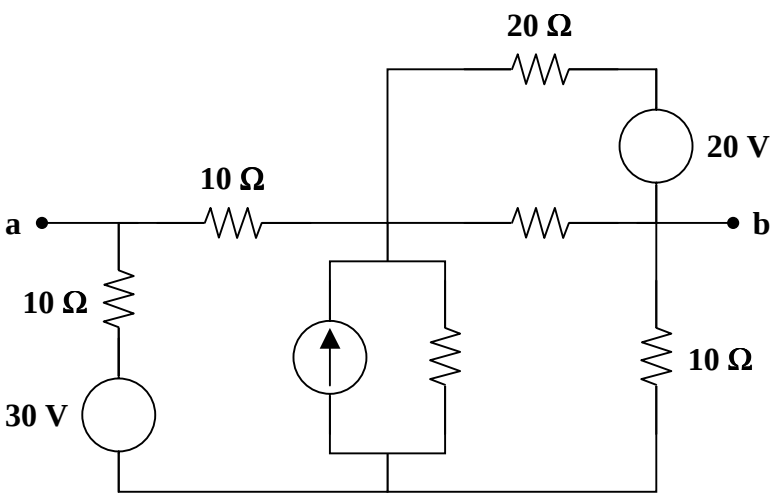
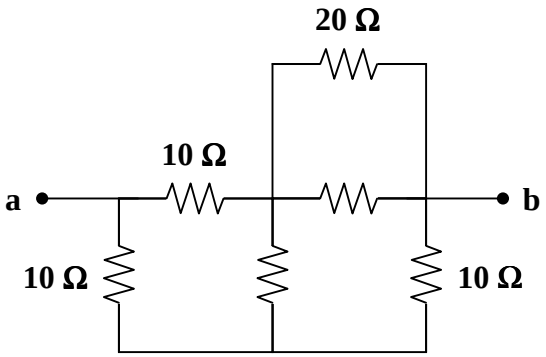


Figure 4.1

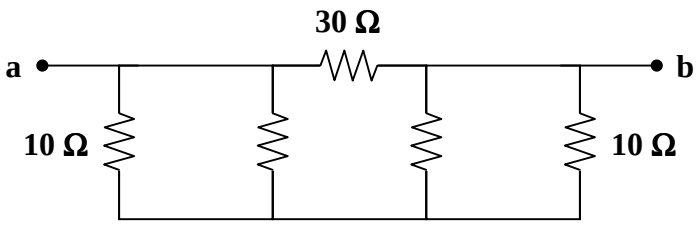
To find R_{Th}, consider the circuit in Fig. (a).



(a)

where $20 \parallel 20 = 10\,\Omega$.

Now, transform the wye subnetwork to a delta as shown in Fig. (b).

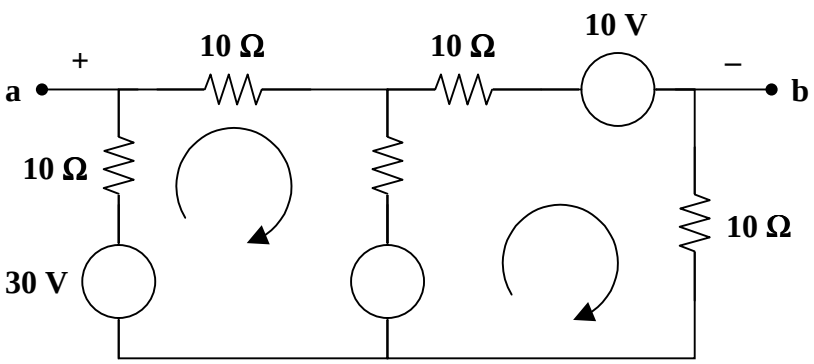


(b)

where $10 \parallel 30 = 7.5\,\Omega$.

Hence, $R_{Th} = R_{ab} = 30 \parallel (7.5 + 7.5) = 30 \parallel 15 = \mathbf{10\,\Omega}$

To find V_{Th}, we transform the 20-V and the 5-V sources to obtain the circuit shown in Fig. (c).



(c)

For loop 1,

$$-30+50+30i_1-10i_2=0$$
$$-2=3i_1-i_2 \qquad (1)$$

For loop 2,

$$-50-10+30i_2-10i_1=0$$
$$6=-i_1+3i_2 \qquad (2)$$

Solving (1) and (2),

$$i_1=0\ \text{A} \quad \text{and} \quad i_2=2\ \text{A}$$

Applying KVL to the output loop,

$$-V_{ab}-10i_1+30-10i_2=0$$
$$V_{ab}=10\ \text{V}$$

$$V_{Th}=V_{ab}=\underline{\mathbf{10\ V}}$$

Problem 4.12 Find the Thevenin equivalent as seen by R in Figure 4.1.

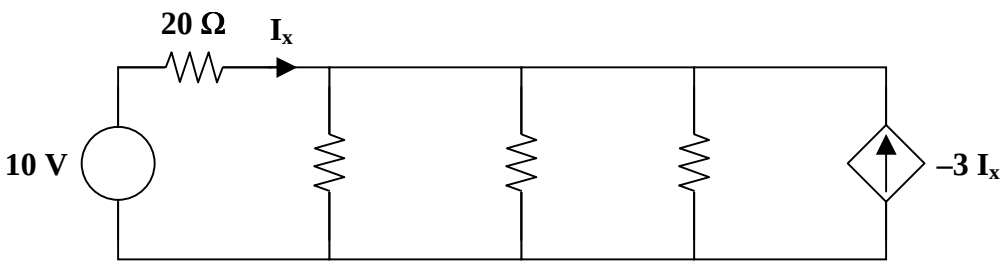
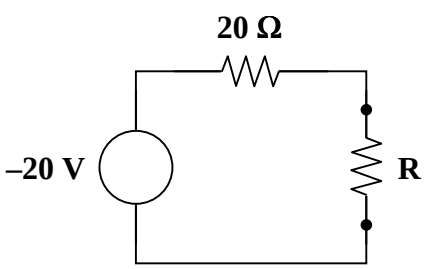


Figure 4.1

Therefore, **the Thevenin equivalent circuit is as follows**.



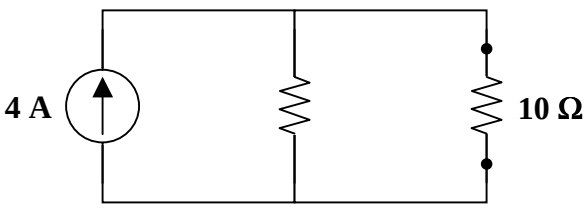
NORTON'S THEOREM

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current at the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Problem 4.13 Find the Norton equivalent of the circuit in Figure 4.1.

In Problem 4.9, $V_{oc} = 40$ volts, $I_{sc} = 4$ amps, and $R_{Th} = 10$ ohms.
Recall that $R_N = R_{Th}$; so $R_N = 10$ ohms.

Therefore, **the Norton equivalent circuit is as shown**.



Problem 4.14 *[4.41]* Given the circuit in Figure 4.1, obtain the Norton equivalent as viewed from terminals:

(a) a-b (b) c-d

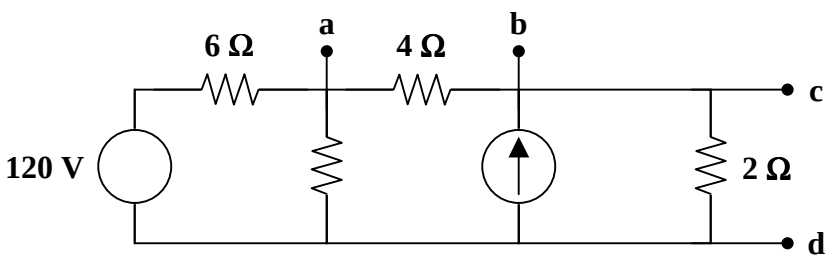
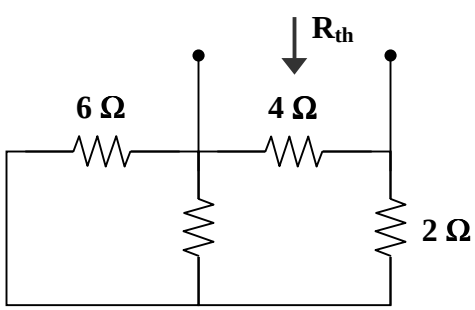


Figure 4.1

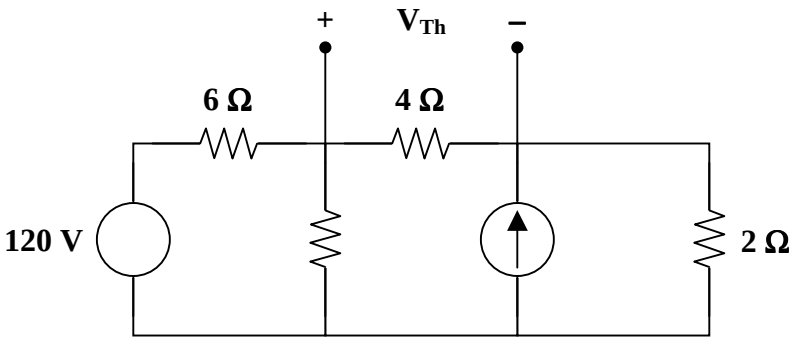
(a) From the circuit in Fig. (a),



(a)

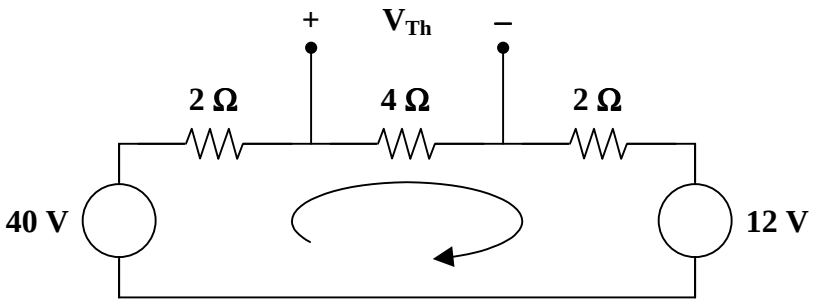
$$R_N = 4 \parallel (2 + 6 \parallel 3) = 4 \parallel 4 = \underline{\mathbf{2\ \Omega}}$$

For I_N or V_{Th}, consider the circuit in Fig. (b).



(b)

After some source transformations, the circuit becomes that shown in Fig. (c).



(c)

Applying KVL to the circuit in Fig. (c),

$$-40 + 8i + 12 = 0 \longrightarrow i = 7/2$$

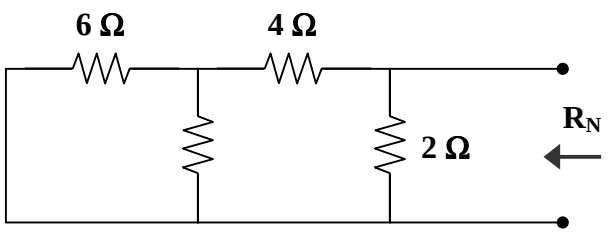
Hence,

$$V_{Th} = 4i = 14\ \text{V}$$

and

$$I_N = V_{Th}/R_N = 14/2 = \underline{\mathbf{7\ A}}$$

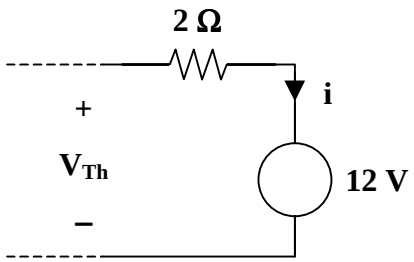
(b) To get R_N, consider the circuit in Fig. (d).



(d)

$$R_N = 2 \parallel (4 + (6 \parallel 3) = 2 \parallel 6 = \underline{\mathbf{1.5\ \Omega}}$$

To get I_N, the circuit in Fig. (c) applies except that it needs slight modification as in Fig. (e).



(e)

$$i = (40 - 12)/(2 + 4 + 2)\ 7/2 \quad \text{and} \quad V_{Th} = 12 + 2i = 19$$
$$I_N = V_{Th}/R_N = 19/1.5 = \underline{\mathbf{12.667\ A}}$$

Problem 4.15 Find the Norton equivalent of the circuit in Figure 4.1.

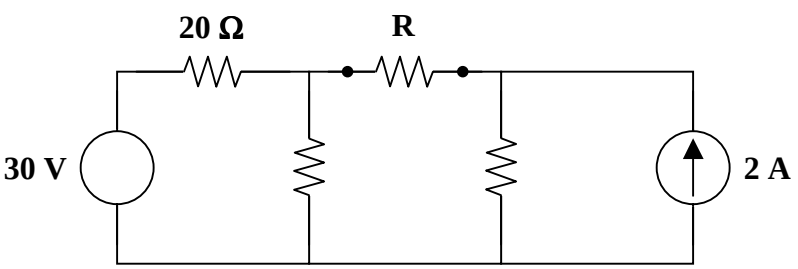
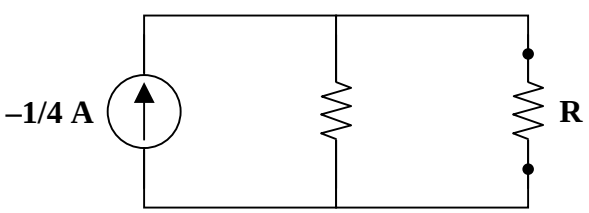


Figure 4.1

Therefore, **the Norton equivalent circuit is as follows**.



MAXIMUM POWER TRANSFER

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen by the load ($R_L = R_{Th}$).

Problem 4.16 Given the circuit in Figure 4.1, complete the following table.

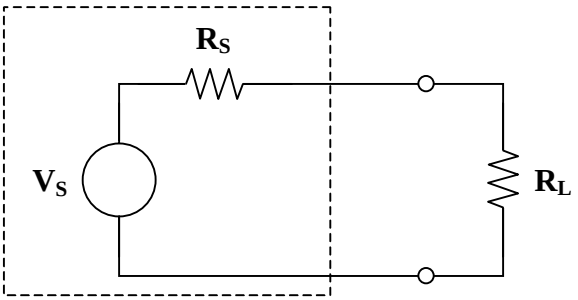


Figure 4.1

	V_S (volts)	R_S (ohms)	R_L (ohms)	Power of R_L (watts)
Condition #1	20	10		
Condition #2	V	10		
Condition #3	20		10	
Condition #4	20			

Let p be the power of the load resistor, R_L. Then, $p = \left(\frac{V_S}{R_S + R_L}\right)^2 R_L$.

We want to find the maximum power transferred to the load. The maximum power is transferred to the load when the load resistance is equivalent to the Thevenin resistance as seen by the load.

In this case, $R_L = R_S$ and $p = \frac{V_S^2}{4R_L}$

Condition #1 : $R_L = R_S = \underline{\mathbf{10\ \Omega}}$ $\qquad p = \frac{(20)^2}{(4)(10)} = \frac{400}{40} = \underline{\mathbf{10\ watts}}$

Condition #2 : $R_L = R_S = \underline{\mathbf{10\ \Omega}}$ $\qquad p = \frac{V^2}{(4)(10)} = \underline{\frac{\mathbf{V^2}}{\mathbf{40}}\ \mathbf{watts}}$

Condition #3 : $p = \left(\frac{20}{R_S + 10}\right)^2 (10)$

Clearly, for maximum power, $R_S = \underline{\mathbf{0\ \Omega}}$. *{with R_L fixed, making R_s as small as possible maximizes the power to R_L}*

Then, $p = \left(\frac{20}{10}\right)^2 (10) = (2^2)(10) = \underline{\mathbf{40\ watts}}$.

Condition #4 : For maximum power, $R_S = \underline{\mathbf{0\ \Omega}}$ and $R_L = \underline{\mathbf{0^+\ \Omega}}$.

Then, $p = \dfrac{20^2}{(4)(0^+)} = \underline{\infty^- \textbf{ watts}}$.

Problem 4.17 *[4.59]*

(a) For the circuit in Figure 4.1, obtain the Thevenin equivalent at terminals a-b.

(b) Calculate the current in $R_L = 8\,\Omega$.

(c) Find R_L for maximum power deliverable to R_L.

(d) Determine that maximum power.

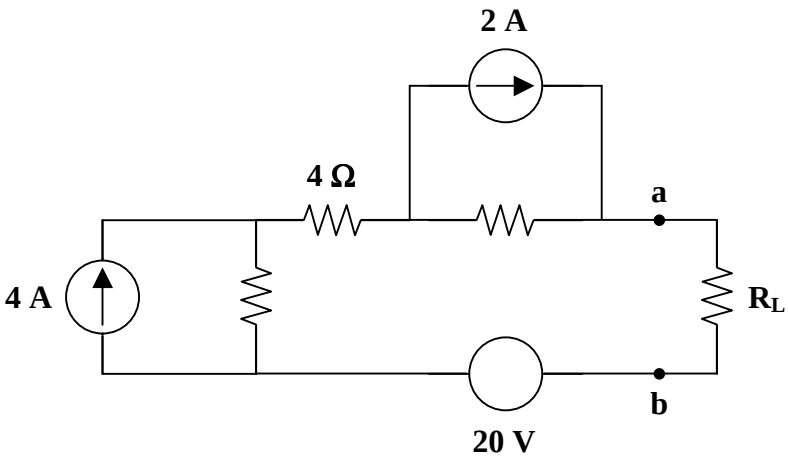
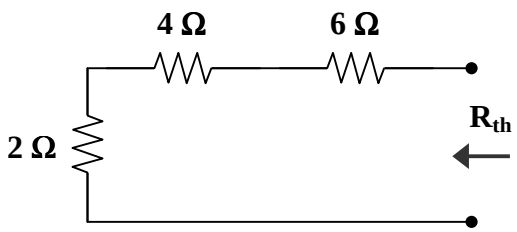


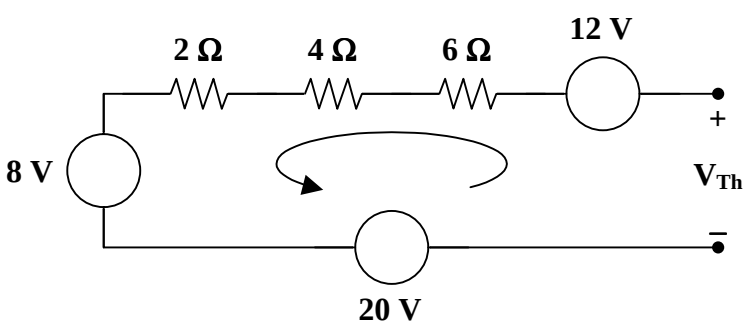
Figure 4.1

(a) R_{Th} and V_{Th} are calculated using the circuits in Fig. (a) and (b) respectively.



(a)

From Fig. (a), $R_{Th} = 2 + 4 + 6 = \underline{\textbf{12 Ω}}$



(b)

From Fig. (b),

$$-V_{Th} + 12 + 8 + 20 = 0$$

$$V_{Th} = \underline{\mathbf{40\ V}}$$

(b) $$i = \frac{V_{Th}}{R_{Th} + R} = \frac{40}{12 + 8} = \underline{\mathbf{2\ A}}$$

(c) For maximum power transfer,

$$R_L = R_{Th} = \underline{\mathbf{12\ \Omega}}$$

(d) $$p = \frac{V_{Th}^{\ 2}}{4R_{Th}} = \frac{(40)^2}{(4)(12)} = \underline{\mathbf{33.33\ watts}}$$

Problem 4.18 Using Figure 4.1, what values of R_S and R_L result in maximum power delivered to the load? What is the power absorbed by the load?

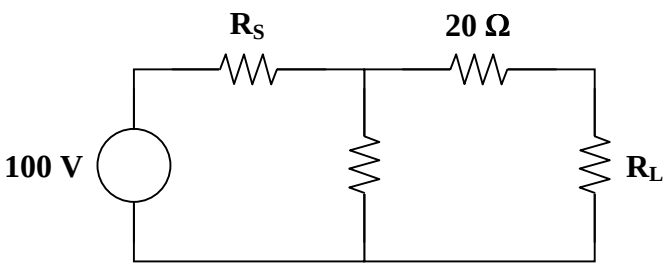


Figure 4.1

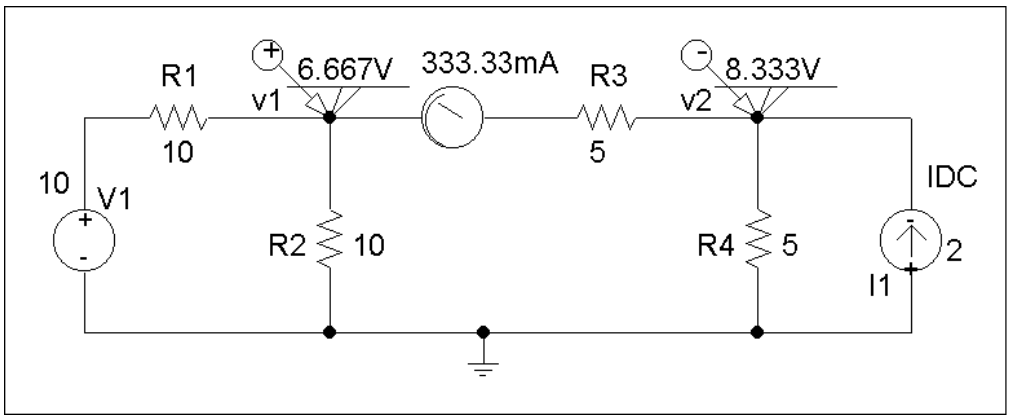
$$R_S = \underline{\mathbf{0\ \Omega}}$$

$$R_L = \underline{\mathbf{20\ \Omega}}$$

$$p_{RL} = \underline{\mathbf{125\ watts}}$$

VERIFYING CIRCUIT THEOREMS WITH PSPICE

Problem 4.19 Solve Problem 4.1 using PSpice.



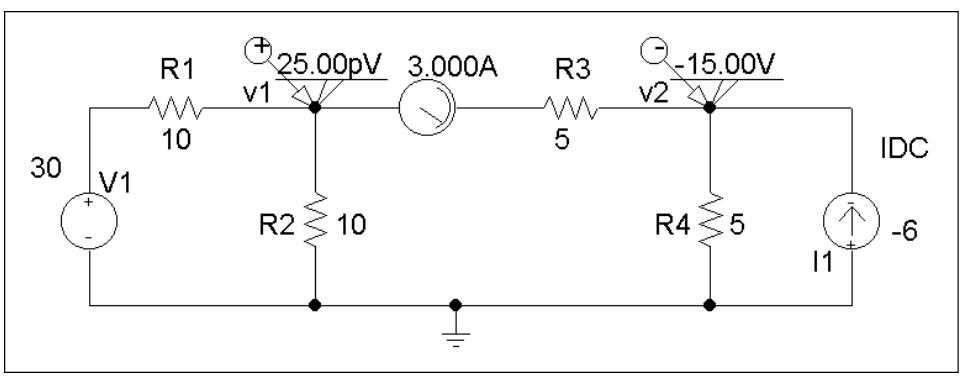
A voltage source in PSpice needs a value to simulate rather than performing a symbolic simulation. So, let $K = 1$. Then, $V_x = -5/3$ volts and $I_x = -1/3$ amps. Clearly, V_x is the voltage across R3 and I_x is the current flowing through R3 from left to right. Thus, for **K = 1**,

$$V_x = v_1 - v_2 = 6.667 - 8.333 = -1.667 = \mathbf{-5/3\ V}$$

and $I_x = V_x/5 = -1/3$ amps or $I_x = \mathbf{-333.333\ mA}$.

These answers agree with the answers obtained in Problem 4.1 given that $K = 1$.

Problem 4.20 Solve Problem 4.3 using PSpice.



Clearly, V_x is the voltage across R3 and I_x is the current flowing through R3 from left to right. Thus, $V_x = v_1 - v_2 = (25 \times 10^{-12}) - (-15) = 15$ V and $I_x = V_x/5 = \mathbf{3\ A}$.

This answer agrees with the answer obtained in Problem 4.3.